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## Analysis of Two Definitions of the Mole That Are in Simultaneous Use, and Their Surprising Consequences

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plural word for atoms, molecules, ions, or electrons. For discrete elementary entities, the "amount" of entities is the *number* of the entities; for instance, 1 mol electrons =  $6.022 \times 10^{23}$  electrons. In short, when applied to elementary entities, the mole must be a number. Most textbooks initially recite the official definition of the mole but later implicitly use the number definition when referring to moles of electrons or of ions such as  $H^+(aq)$ . While most practicing chemists ignore the subtleties of the official definition, its ambiguity trying to link a continuum substance concept with a number of countable entities must confuse students. Like previous authors, we argue that a substance essentially equals the molecules of which it consists, and therefore, "amount of substance" is equal to the "number of molecules". In this framework, the mole is a number unit analogous to dozen or percent. The number definition has interesting consequences that have not been exploited due to its unofficial status: If 1 mol =  $6.022 \times 10^{23}$ , then the gas and Boltzmann constants are equal,  $R = k_{\rm B}$ . Since 1 mol =  $6.022 \times 10^{23}$  for electrons, the Faraday constant equals the elementary charge,  $F = 96,485 \text{ C}/6.022 \times 10^{23} = 1.602 \times 10^{-19} \text{ C} = e$ , and 1 eV = 96.5 kJ/mol. These unifying equalities will make it easier to learn and teach chemistry.

"mole of electrons"

**KEYWORDS:** First-Year Undergraduate/General, High School/Introductory Chemistry, History/Philosophy, Nomenclature/Units/Symbols, Misconceptions/Discrepant Events

### INTRODUCTION

The mole is a central concept in chemistry: Molar masses are used throughout chemistry, concentrations in mol/L are widely used in various equilibrium calculations in general chemistry, and many quantities in physical chemistry, including the chemical potential, are "molar", with "/mol" in the units. Experienced chemists sometimes express bewilderment why students have trouble<sup>1-3</sup> understanding the mole. We conclude here that the widely acknowledged confusion<sup>1,3–9</sup> is created by the use of two related but distinct definitions of the mole in chemistry.<sup>1,10,11</sup> Most textbook authors initially pay lip service to the official definition,<sup>12</sup> which is linked to the international system of units ("SI"), sanctioned by the IUPAC, and requires the mole not be a number; however, like many practicing chemists,<sup>1,10</sup> when solving problems the same books often revert to a simpler usage of the mole as a number, though this is not rigorously introduced due to its unofficial status. This inconsistency must confuse students learning about the mole.

is followed by a singular word for a substance (e.g., "water"), or a

According to the official IUPAC definition,<sup>13</sup> mole is a *unit*, associated with a quantity called "amount of substance" that is given the symbol n. Being based on a continuum view of substance,<sup>9</sup> the official mole is not a number;<sup>2,3,5,6,9,14</sup> otherwise, it could not be an independent base unit in the international system of units (SI). $^{6,9,15}$  The official definition is cited in many textbooks when the mole is introduced,  $1^{6-21}$  but in later sections of the same books<sup>16–21</sup> and in actual chemical practice, one mole of atoms, molecules, or electrons is taken to be equal to a number, specifically 6.022 140 76  $\times$  10<sup>23</sup>,<sup>13</sup> of atoms, molecules, or electrons; in short

 $1 \text{ mol} = 6.022 \times 10^{23}$ (1)

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written for brevity with only four significant figures, which are sufficient for conceptual and undergraduate numerical purposes. Equation 1 is expressed (as a sentence) in at least one textbook<sup>22</sup> and has been shown prominently in online tutorials<sup>23,24</sup> explaining the mole.

The existence of several definitions of the mole has been pointed out before, 3,4,8,10,25 but often the simple number definition of eq 1 was considered as objectionable, and it has therefore not been written as an equation in the peer-reviewed literature.<sup>5,8,14</sup> We document that, though often unacknowledged, the number definition is in active use in all general chemistry textbooks as well as in chemical practice. We compare the two definitions and discuss how one can determine which one is used in a specific instance. Previous discussions<sup>4,8,25</sup> of the mole have usually focused on the simple case of pure substances, such as "mole of tin", where a distinction from "mole of tin atoms" seems almost unnecessary. By highlighting the example of the "mole of electrons",<sup>7</sup> widely used (>200,000 Google hits) in particular in electrochemistry and found in every general chemistry text that we have checked, 16-21 we point out that the mole as the unit of uncountable amount of substance is too limited in its applicability, since there is no "electron substance" distinct from the countable electrons; the mole must be a number here, 1 mol electrons =  $6.022 \times 10^{23}$  electrons. It is proposed that with the universally accepted atomistic view of matter, "amount of substance" can be considered as equal to "number of entities"<sup>1,7</sup> and the text of the official definition can be reconciled with eq 1. Furthermore, we demonstrate that the practical numerical definition (eq 1) makes various constants and units in physical chemistry equal to their counterparts in physics, thus simplifying learning for students of these subjects.

### ANALYSIS AND DISCUSSION

### The Official Definition of the Mole

According to the most recent definition by IUPAC and the International System of Units  $(SI)^{12,13}$ 

"The mole, symbol mol, is the SI unit of amount of substance.

One mole contains exactly  $6.02214076 \times 10^{23}$  elementary

entities. This number is the fixed numerical value

of the Avogadro constant,  $N_A$ , when expressed in mol<sup>-1</sup>

and is called the Avogadro number. The amount

of substance, symbol *n*, of a system is a measure

of the number of specified elementary entities. An

elementary entity may be an atom, a molecule, an ion,

an electron, any other particle or specified group of particles."

(2)

The core phrase, to four significant figures, is

'1 mole contains 6.022 
$$\times$$
 10<sup>23</sup> elementary entities" (2')

Definition 2 is simpler than the corresponding text in force before 2019, but definition 2' and the statement that the mole is the SI unit of amount of substance has not changed. We will discuss the implications of this definition and compare it with the definition in eq 1 used in practice by many chemists.

### "Mole of a Substance" versus "Mole of Entities": Singular vs Plural

In the official definition 2, mole is the unit of "amount of substance", an ill-defined quantity.<sup>1,7,8,10,26</sup> It is easy to verify that any conventional term for a substance ("water", "butter", "air", "oxygen", "tin") is singular in form. This matches with the observation that, in standard English, "amount of" should be followed by a singular, uncountable word for a substance (water, oxygen, oil, etc.), not by discrete countable items (apples, atoms, molecules, cats), which would instead be preceded by "number of".<sup>27</sup> An expression like "amount of cats" is poor English. Note that we use countable in the mathematical sense (which includes "countably infinite"), i.e., meaning countable in principle. A rare specific example in IUPAC's explanations of the redefinition of the mole, "amount of water",<sup>13</sup> conforms to the grammatical requirement that "amount of" be followed by a singular term for substance. Upon reflection, it becomes clear that this concept of substance embodies an intuitive, pre-20th century continuum view of substance as not countable, like volume. The IUPAC Technical Report on the recent redefinition of the mole indeed confirms explicitly that "the conventional definition relies on continuum physics."

However, in most textbooks and in chemical practice, the word "mole of" is often followed by a *plural* term for *discrete* entities, such as specific atoms, molecules, or electrons.<sup>7,16–21</sup> From definition 2′, the number corresponding to a mole is 6.022  $\times 10^{23}$ , so if mole refers to countable entities, it follows that

$$1 \text{ mol}(e \text{ of}) \text{ entities} = 6.022 \times 10^{23} \text{ entities}$$
 (3)

Leaving out "entities" on both sides of eq 3, we have again eq 1. It is important to acknowledge this critical result: *the "amount" of countable entities is simply their number*. An online search of "amount of electrons" confirms that it is commonly equated to the number of electrons. There is no question that

1 mol electrons =  $6.022 \times 10^{23}$  electrons

This means that the mole *must* be a number when applied to countable discrete entities.

We can confirm this central conclusion by paraphrasing the question "What is the "amount" of electrons transferred when 32.7 grams of zinc metal were oxidized to  $Zn^{2+}$ ?" in a way that avoids the grammatically questionable use of "amount" followed by a plural word for discrete entities. We cannot start this question with "How much...", which would be the appropriate wording when asking about the amount of a substance like water or air. Rather, we have to ask "How many electrons were transferred when 32.7 grams of zinc metal were oxidized to  $Zn^{2+}$ ?", and the answer is clearly a number. The answer is also "1 mol", which confirms that the mole must be a number in this context.

### Substance versus Discrete Entities

The concept of substance in the conventional macroscopic, continuum view implies that a substance should fill a certain connected volume mostly free of other entities. One can argue that with large numbers of tightly packed entities, e.g., in "amount of peas" or "amount of water molecules", the use of "amount of" followed by a plural word is less jarring. However, the tight-packing requirement is not fulfilled in many cases where chemists need the mole concept. For instance, Figure 1 is a depiction of  $0.9963 \times 10^{-5}$  attomoles of Na<sup>+</sup> ions, i.e., six Na<sup>+</sup> ions, in a solvent. The figure highlights that referring to these discrete ions as "Na<sup>+</sup> ion substance" is extremely forced and that



**Figure 1.** Schematic showing  $0.9963 \times 10^{-5}$  attomoles of Na<sup>+</sup> ions in aqueous NaCl solution. The "Na<sup>+</sup> ion substance" implied by the substance-based IUPAC definition is shown shaded within the dashed circles, which represent an ill-defined cutoff. The "amount" of Na<sup>+</sup> ions, properly called the number of Na<sup>+</sup> ions, shown here is  $0.9963 \times 10^{-5}$  attomoles = 6.

the mole must be a number here since it refers to discrete, countable ions.

Similarly, there is neither a conventional "electron substance" nor a "hydronium substance". Therefore, the substance-based definition of the mole, which works fine for common pure substances like tin, water, or oxygen gas, fails in these important cases. "Mole of electrons" is widely used in electrochemistry, and we have found this terminology in every general chemistry textbook consulted (under "Faraday constant").<sup>16–21</sup> The molarity of hydronium ions is of course a central quantity in acid–base chemistry. The fact that "electrons" or "hydronium ions" are plural words is the telltale sign that the "amount" of such entities is their number: 1 mol hydronium ions =  $6.022 \times 10^{23}$  hydronium ions. The instructive analogy of the mole as a number with the (baker's) dozen is discussed in the Supporting Information, as is evidence that the mole followed by a chemical formula also implies the number definition.

#### Ambiguity in the Official Definition of the Mole

Both concepts of the mole, as the unit of uncountable "amount of substance" (continuum view<sup>9</sup>) and as a countable "number of specified elementary entities" (atomistic view), seem to appear in the IUPAC definition 2, and it is not clear that they have been logically reconciled. Figure 1 showing disjointed "Na<sup>+</sup> ion substance" demonstrates that a definition of a substance in terms of "specified elementary entities" sometimes cannot conform to IUPAC's underlying continuum-physics<sup>9</sup> concept of matter.

In the IUPAC definition and technical report,<sup>9,13</sup> specific examples of "mole of..." are few and it therefore remains uncertain whether the IUPAC allows "mole of..." to be followed by a plural word for specified entities ("water molecules", "electrons", etc.). If so, it would need to be explained how this usage could be reconciled with the mole not being a number, given that the "amount" of countable entities is necessarily the number of those entities.

Reading the central phrase "One mole contains exactly  $6.022\ 140\ 76 \times 10^{23}$  elementary entities" of definition 2, one needs to ask "One mole of what?" According to the third sentence of the definition, elementary entities need to be specified. This gives us: "One mole of elementary entities contains exactly  $6.022\ 140\ 76 \times 10^{23}$  elementary entities", which can be simplified to "One mole of elementary entities is equal to

exactly  $6.02214076 \times 10^{23}$  elementary entities", since "contains" can be replaced with "is equal to" for number words, as shown in the Supporting Information. Striking "of elementary particles" on both sides of the equality finally gives "one mole is equal to exactly  $6.02214076 \times 10^{23}$ ", the number definition.

While the number aspect is clearly in the foreground in the text of the official definition, the continuum concept of substance is emphasized in the accompanying IUPAC report,<sup>9</sup> since it is the underpinning of considering "amount of substance" as a SI base quantity distinct from the number of particles.<sup>2,3,5,9,14</sup> According to the SI Brochure,<sup>15</sup> quantities that "have the nature of a count", i.e., are integer numbers, have the "unit one" and "cannot be described in terms of the seven base quantities of the SI".<sup>15</sup> Thus, if the mole were a number, it would be a multiple of the unit one and could not be a SI base unit.<sup>9,15</sup> Accordingly, the IUPAC has made it clear<sup>6,9</sup> that it views it as incorrect to consider the mole as a number; this is documented with quotes in the Supporting Information.

### The Mole as a Number Works Well in Practice

Defenders of the official continuum concept of substance have asserted<sup>2,3,5,6,9,14</sup> that considering the amount of substance and the mole as a number,<sup>1</sup> as in eq 1, is "incorrect"<sup>5</sup> or "controversial".<sup>14</sup> Nevertheless, no one has been able to argue convincingly that eq 1 (or equivalently eq 3) does not work in chemical practice,<sup>1</sup> and it fits well into the framework of our modern atomistic world view.<sup>10,28</sup> Freeman's<sup>29</sup> claim that the mole is not a number because "The flask contains 6.022p23 of water" is "neither clear nor logical" falls apart if "molecules" is appended after "water", as is commonly done: 1 mol water molecules =  $6.022 \times 10^{23}$  water molecules. In the following, uses of the number definition of the mole in the literature, and of its intriguing consequences, are documented.

**The Mole as a Number in Textbooks.** A textbook<sup>22</sup> and an online source<sup>24</sup> introduce the mole as a number equivalent to eq 1 and thus keep the presentation simple. Others<sup>16–21</sup> initially follow the official definition and discuss only moles of pure substances, avoiding moles of ions, atoms in molecules, or electrons. However, later in the context of Faraday's constant, invariably these texts cannot avoid referring to "1 mol of electrons", implying eq 1 and violating their own substance-based definition. Students would be better served if the simple number definition in eq 1 was presented as an option from the start.

Some books introduce aspects of both definitions of the mole. For instance, Atkins et al.<sup>16</sup> on one hand state that the mole is analogous to a dozen (i.e., a unitless number) but on the same page describe it as the unit of "amount of substance". The definition of the mole in the same text, "1 mol of objects means  $6.022 \times 10^{23}$  of those objects", raises the question what "means" means. Is "means" equivalent to "is equal to"? (Our analysis for the dozen in the Supporting Information strongly suggests that it is.) The simple eq 1 would provide more clarity than an ambiguous phrase. Giunta's analysis of the introduction of the mole in several other textbooks, in particular Tro's,<sup>30</sup> has revealed similar ambiguities.<sup>8</sup> Examples of the mole as a number in the specialized literature are discussed in the Supporting Information.

# Convenient Equalities from the Number Definition of the Mole

The simple numerical equality (eq 1), which despite official disapproval is widely considered as equivalent to the official

definition by practicing chemists,<sup>9</sup> results in a simplifying unification of pairs of "equivalent" quantities or units in physical chemistry and physics.

Equality of Faraday's Constant and the Elementary Charge. It is universally accepted that Faraday's constant F is the magnitude of electric charge per amount ("per mole") of electrons<sup>16–21</sup>

$$F = 96,485 \text{ C/mol (electrons)}$$
(4)

Electrons are countable, and there is no alternative to the equality

$$1 \text{ mol (electrons)} = 6.022 \times 10^{23} \text{ (electrons)}$$
(5)

Combining these two simple eqs 4 and 5, one finds

$$F = \frac{96,485 \text{ C}}{1 \text{ mol (electrons)}} = \frac{96,485 \text{ C}}{6.022 \times 10^{23} \text{ (electrons)}}$$
$$= \frac{1.602 \times 10^{-19} \text{ C}}{1 \text{ (electron)}} = e \tag{6}$$

This shows that the Faraday constant *F* is *equal* to the charge per electron, which is the elementary charge *e*.

**Potential Equalities of Other Natural Constants.** It is widely accepted that the gas constant is  $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$  and the Boltzmann constant  $k_{\text{B}} = 1.38 \times 10^{-23} \text{ J/K}$ . If  $1 \text{ mol} = 6.022 \times 10^{23}$ , we have

$$R = 8.31 \frac{J}{\text{mol K}} = \frac{8.31 J}{6.022 \times 10^{23} \text{ K}} = 1.38 \times 10^{-23} \frac{J}{\text{K}} = k_{\text{B}}$$
(7)

So if eq 1 holds, the gas constant is not just equivalent but *equal* to the Boltzmann constant.<sup>6</sup> Adopting this equality greatly simplifies the transition between statistical mechanics, typically developed using  $k_{\rm B}$ , and classical thermodynamics using R; see the Supporting Information for examples. The conventional relations with the Avogadro constant  $N_{\rm A} = 6.022 \times 10^{23}$ /mol, namely,  $R = N_{\rm A} k_{\rm B}$  and  $F = N_{\rm A} e$ , still hold, since 1 mol = 6.022 ×  $10^{23}$  results in

$$N_{\rm A} = 6.022 \times 10^{23} / \text{mol} = 6.022 \times 10^{23} / 6.022 \times 10^{23} = 1$$
(8)

(Note that the Avogadro *number* is still  $6.022 \times 10^{23}$ .) 1 mol =  $6.022 \times 10^{23}$  is implicit in the widely accepted equalities

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$
 (9a)

$$1 \text{ eV} = 96,485 \text{ J/mol}$$
 (9b)

(see, for instance, Atkins et al.,<sup>31</sup> Table F5). Indeed, setting eqs 9a and 9b equal gives 1 mol = 96,485 J/1.602 ×  $10^{-19}$  J = 6.022 ×  $10^{23}$ . We also find that eq 1 results in the units g/mol and dalton being not merely equivalent, but equal (to within the relative uncertainty of < $10^{-9}$  implied in the new definition of the mole):<sup>9</sup>

$$lg/mol = 1g/6.022 \times 10^{23} = 1.66 \times 10^{-24}g = 1 u$$
  
= 1 Da (10)

The same applies for the underlying physical quantities:

$$molar mass = molecular mass$$
 (11)

For example, for H<sub>2</sub>O, the molar mass is  $18 \text{ g/mol} = 18 \text{ g/6.022} \times 10^{23} = 3 \times 10^{-23} \text{ g} = 18 \text{ Da} = \text{the mass of an H<sub>2</sub>O molecule.}$ 

# Amount of Substance and Number of Molecules versus Mass

The concept of "amount of substance", a quantity in practice often referred to as "number of moles",<sup>8</sup> is rooted in an outdated continuum view of matter.<sup>4,9,32</sup> Nevertheless, the IUPAC definition<sup>8,33</sup> does acknowledge that amount of substance *n* is strictly proportional to the number *N* of entities,<sup>6,8,34–36</sup> specifically

$$n = (1/N_{\rm A})N\tag{12}$$

with the fixed Avogadro constant  $N_A$ . Conceptual diagrams positioning the amount of substance *n* symmetrically between mass *m*, volume *V*, and  $N^{3,4,37}$  suggest that amount of substance is as closely related to mass and volume of substance as to the number of atoms/molecules that constitute it. In the modern atomistic worldview, this seems misguided. While synergistic effects such as intermolecular interactions often make a substance more than just a collection of its constituent elementary entities, e.g., imparting surface tension to liquid water, these interactions do not change the *amount* of the substance. A consideration of synergistic effects may underlie the terminology "liquid water *consists of* water molecules", but in terms of the amount, a mole of water *is* 6.022 × 10<sup>23</sup> water molecules.

This close relation between n and N is reflected in their universal fixed proportionality, eq 12. A substance *has* mass but is not *equal* to its mass, just like mass usually takes volume but is not equal to its volume. The proportionality factors, molar mass between amount of substance and mass, and density between mass and volume, are not fixed and universal, which indicates that amount of substance, mass, and volume are distinct quantities. However, no fundamental distinction exists between amount of substance and the number of molecules, according to the universal proportionality in eq 12. These relations are represented schematically in Figure 2 (where "of substance" should be included in the label of the quantity V, since volume exists in the absence of substance). Since the coefficient  $N_A$ 



**Figure 2.** Schematic of the relations between mass (of substance) *m*, volume of substance *V*, and number of atoms/molecules/ions *N* or equivalently amount of substance *n*, for a pure substance. "Counting by weighing" is discussed in the Supporting Information.

between the number of particles N and amount of substance n is not variable, the two quantities can share a box in the figure.

Amount of Substance is the Number of Molecules. The proportionality between N and n is universally given by eq 12. Indeed, in the accepted atomistic worldview, it is unnecessary to make a distinction between the amount of a substance and the number of atoms or molecules that compose it.<sup>6,8,34-36</sup> (This is analogous to the size of a flock of sheep being equal to the number of sheep.) Accordingly, the IUPAC definition<sup>8,33</sup> states that the amount of substance "has to be treated almost identically with the number of entities", and the recent revision has moved the mole from a mass-based closer to a number-based definition.<sup>9</sup> In the context of the mole of electrons, the artificial "electron substance"<sup>7</sup> implied by the IUPAC definition is just a countable collection of electrons in a matrix, analogous to the "sodium ion substance" in Figure 1. There are no synergistic interactions between these elementary entities, which makes it particularly clear that the amount of "electron substance" is simply the number of electrons, n = N. A few articles on "amount of substance"26,36 reviewed in the Supporting Information have come to the same conclusion.

Mathematically, accepting that 1 mol electrons =  $6.022 \times 10^{23}$  electrons directly leads to eq 1, which in turn results in  $N_A$  = 1, see eq 8; then, eq 12 simplifies to

$$n = N \tag{13}$$

As a result, the boxes for *n* and *N* in the conceptual diagram for a pure substance<sup>3,4,37</sup> can be merged completely, see Figure 2, simplifying chemistry conceptually. Just like the number of school children in a class can be given as 24 or 2 dozen, the number of electrons transferred in an electrochemical process can be given as  $n = 12.044 \times 10^{23}$  or n = 2 mol. Considering *n* as the number of atoms, ions, molecules, or electrons is versatile: as we have pointed out, there is no conventional "electron substance" whose amount could be quantified, but the "number of electrons" is well-defined and conceptually simple.

In other words, if one acknowledges that, atomistically, the amount of substance n is equal to the number of particles N constituting the substance,<sup>36</sup> the text of the official definition 2 of the mole can largely be reconciled with the number definition in eq 1. This corresponds to bridging the fine conceptual distinctions between "1 mol of water" and "1 mol of water" molecules" and between "contains", "consists of", and "is". Amount of substance is then a number (of particles), and mole is a very useful number unit, analogous to percent (%), dozen (dz), or million (M); just as

$$1\% = 0.01$$
  
 $1 dz = 12$   
 $1 M = 1 \times 10^{6}$ 

we have

 $1 \text{ mol} = 6.022 \times 10^{23}$ 

This view appears to be equivalent to Emerson's,<sup>36</sup> and to Baranski's proposal<sup>7</sup> of interpreting amount of substance as the "quantity of microentities", given that the *quantity* of countable entities is conventionally called the *number* of such entities.

### IMPACT ON TEACHING

Consistently presenting the mole as a number, rather than first introducing it as the unit of a vaguely defined quantity but later treating it "unofficially" as  $6.022 \times 10^{23}$ , will greatly reduce

confusion when students learn about the mole. The text of the official definition 2, on its face, appears to equate a mole of entities and  $6.022 \times 10^{23}$  entities; it is ambiguous as to whether the mole refers to uncountable substance or countable entities. Given such ambiguities in the official definition, instructors should be allowed to replace "contains" with "is equal to": "One mole is equal to  $6.022 \ 140 \ 76 \times 10^{23}$  [elementary entities]". That this replacement is permissible for number words has been demonstrated in the Supporting Information, where examples of eliminated conceptual problems listed by Furio et al.<sup>2</sup> are also given. The unifying "new equalities" F = e,  $R = k_{\rm B}$ , and 1 Da = 1 g/mol, which like 1 eV = 96,485 J/mol result from the properly acknowledged eq 1, may initially confound some professors long accustomed to the old distinctions but will simplify learning for new students of chemistry.

### CONCLUSIONS

We have documented that two related but distinct definitions of the mole, as a basic SI unit and as a number, have been in use, often in the same text, and pointed out their specific, previously overlooked consequences. According to the official IUPAC and SI definition, the mole is the unit of "amount of substance" and may not be considered as a number. As a consequence, given that there is no electron or hydronium substance, this definition is not really applicable to countable electrons or hydronium ions and thus fails to fully serve chemists' needs. This problem would be resolved if chemistry was freed from the complications of an outdated continuum concept of substance and the mole was defined simply as 6.022...  $\times$  10<sup>23</sup>, as it had been in many textbooks before 1971.8 The literature has not shown any convincing evidence that this number definition falls short in practice; for countable entities that are dispersed, for instance in a solvent, and therefore do not form a substance distinct from the discrete entities themselves, the mole *must* be a number: 1 mol Na<sup>+</sup> ions =  $6.022 \times 10^{23}$  Na<sup>+</sup> ions. Analogous equations hold for atoms, molecules, formula units, and electrons. Many chemists have not really accepted the continuum aspects of the official definition and instead still use the mole as a number; indeed, this usage eliminates conceptual problems identified by chemical educators. A simple singular vs plural analysis has shown that all textbooks implicitly use the mole as a number, 1 mol =  $6.022 \times 10^{23}$ , for electrons and ions, even after having initially recited the officially imposed amount-of-substance definition. We have further pointed out that 1 mol =  $6.022 \times$ 10<sup>23</sup> has surprising consequences that have been overlooked since the equation was mostly considered as a shortcut and not written down "officially". Important constants as well as some units in physical chemistry and physics are unified: if 1 mol =  $6.022 \times 10^{23}$ , then  $R = k_{\rm B}$  and dalton = g/mol; the equalities F = eand 1 eV = 96,485 J/mol are unconditional. These relations should be considered as welcome simplifications for students learning chemistry. Like others in the literature, we argue that the amount of a substance is the number of atoms or molecules that constitute it. Therefore, "amount of substance" is equal to the conceptually simple "number of entities" and can be given as a number, in dozens, or most conveniently, in moles.

### ASSOCIATED CONTENT

### **Supporting Information**

The Supporting Information is available at https://pubs.acs.org/doi/10.1021/acs.jchemed.9b00467. Discussions of the analogy of the mole with the dozen; that "contains" can mean "is equal to"; whether "amount of substance" is based on the continuum illusion; of evidence that the IUPAC/SI mole is based on continuum physics, cannot be a number, and does not apply to electrons; of counting by weighing; of the elimination of conceptual problems by the number definition; of mole of a chemical formula; of  $R = k_{\rm B}$  in thermodynamics; of chemical education literature about the mole; and of IUPAC's weak arguments for the mole as a base unit (PDF)

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### Notes

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